MORE PLEASURES OF COUNTING

Suppose there is a Petanque tournament with n players that must be arranged into an even number of teams. The teams may be triples or doubles or combinations of both depending upon the tournament format. If the tournament format is *triples* then there should be the maximum number of triples teams. If the tournament format is *doubles* then there should be the maximum number of doubles teams.

Here is a simple way of determining the combinations of teams.

n is the number of players, T is the number of triples teams and D is the number of doubles teams.

TRIPLES FORMAT

- Let $x = \dfrac{n}{6}$ and x is a *rational number* that will be either:

    (i) an integer K. For example, if $n = 48$ then $x = \dfrac{48}{6} = 8 = K$

    (ii) a decimal number with an *Integer part* $(\,Ip\,(x)\,)$ and a *Fractional part* $(\,Fp\,(x)\,)$. For example, if $n = 43$ then $x = \dfrac{43}{6} = 7\dfrac{1}{6} = 7.1666\ldots$ and $Ip\,(x) = 7$ and $Fp\,(x) = \dfrac{1}{6} = 0.1666\ldots$

- If $x = \dfrac{n}{6}$ is an integer *K;* then $D = 0$ and $T = 2K$
- If $x = \dfrac{n}{6}$ is a decimal; then let $K = Ip\,(x) + 1$ and $D = 6K - n$ and $T = 2K - D$.

The total number of teams will be $2K.$

Examples:

1. $n = 48$ then $x = \dfrac{48}{6} = 8$ (integer) and $K = 8$,
   $D = 0$ and $T = 2K = 16$

2. $n = 43$ then $x = \dfrac{43}{6} = 7.1666\ldots$ (decimal) and $K = Ip\,(x) + 1 = 8$,
   $D = 6K - n = 48 - 43 = 5$ and $T = 2K - D = 16 - 5 = 11$

3. $n = 41$ then $x = \dfrac{41}{6} = 6.8333\ldots$ (decimal) and $K = Ip\,(x) + 1 = 7$,
   $D = 6K - n = 42 - 41 = 1$ and $T = 2K - D = 14 - 1 = 13$

Using this method leads to a table with a repeating sequence

Players	Teams		Total
	Triples	**Doubles**	
n	**T**	**D**	**Teams**
13	1	5	6
14	2	4	6
15	3	3	6
16	4	2	6
17	5	1	6
18	6	0	6
19	3	5	8
20	4	4	8
21	5	3	8
22	6	2	8
23	7	1	8
24	8	0	8
25	5	5	10
26	6	4	10
27	7	3	10
28	8	2	10
29	9	1	10
30	10	0	10
31	7	5	12
32	8	4	12
33	9	3	12
34	10	2	12
35	11	1	12
36	12	0	12

Players	Teams		Total
	Triples	**Doubles**	
n	**T**	**D**	**Teams**
37	9	5	14
38	10	4	14
39	11	3	14
40	12	2	14
41	13	1	14
42	14	0	14
43	11	5	16
44	12	4	16
45	13	3	16
46	14	2	16
47	15	1	16
48	16	0	16
49	13	5	18
50	14	4	18
51	15	3	18
52	16	2	18
53	17	1	18
54	18	0	18
55	15	5	20
56	16	4	20
57	17	3	20
58	18	2	20
59	19	1	20
60	20	0	20

Table 1: Combinations of Triples and Doubles teams for a TRIPLES tournament with an even number of teams.

Note that these are not the only combinations, just the combinations with the maximum number of triples teams. For example for $n = 43$ players $K = 8,\ D = 6K - n = 5$ and $T = 2K - D = 11$. Another combination can be found by incrementing K by one, so $K = 9$ giving $D = 6K - n = 11$ and $T = 2K - D = 7$.

DOUBLES FORMAT

- Let $y = \dfrac{n}{4}$ and x is a *rational number* that will be either:

  (i) an <u>integer</u> L. For example, if $n = 48$ then $y = \dfrac{48}{4} = 12 = L$

  (ii) a <u>decimal number</u> with an *Integer part* ($Ip(x)$) and a *Fractional part* ($Fp(x)$). For example, if $n = 43$ then $y = \dfrac{43}{4} = 10\dfrac{3}{4} = 10.75$ and $Ip(x) = 10$ and $Fp(x) = \dfrac{3}{4} = 0.75$

- If $y = \dfrac{n}{4}$ is an integer L; then $T = 0$ and $D = 2L$
- If $y = \dfrac{n}{4}$ is a decimal; then let $L = Ip(x)$ and $T = n - 4L$ and $D = 2L - T$.

The total number of teams will be $2L$.

Examples:

1. $n = 28$ then $y = \dfrac{28}{4} = 7$ (integer) and $L = 7$,
   $T = 0$ and $D = 2L = 14$
2. $n = 43$ then $y = \dfrac{43}{4} = 10.75$ (decimal) and $L = Ip(y) = 10$,
   $T = n - 4L = 43 - 40 = 3$ and $D = 2L - T = 20 - 3 = 17$
3. $n = 30$ then $y = \dfrac{30}{4} = 7.5$ (decimal) and $L = Ip(x) = 7$,
   $T = n - 4L = 30 - 28 = 2$ and $D = 2L - T = 14 - 2 = 12$

Using this method leads to a table with a repeating sequence

Players	Teams		Total
	Doubles	Triples	
n	D	T	Teams
12	6	0	6
13	5	1	6
14	4	2	6
15	3	3	6
16	8	0	8
17	7	1	8
18	6	2	8
19	5	3	8
20	10	0	10
21	9	1	10
22	8	2	10
23	7	3	10
24	12	0	12
25	11	1	12
26	10	2	12
27	9	3	12
28	14	0	14
29	13	1	14
30	12	2	14
31	11	3	14
32	16	0	16
33	15	1	16
34	14	2	16
35	13	3	16

Players	Teams		Total
	Doubles	Triples	
n	D	T	Teams
36	18	0	18
37	17	1	18
38	16	2	18
39	15	3	18
40	20	0	20
41	19	1	20
42	18	2	20
43	17	3	20
44	22	0	22
45	21	1	22
46	20	2	22
47	19	3	22
48	24	0	24
49	23	1	24
50	22	2	24
51	21	3	24
52	26	0	26
53	25	1	26
54	24	2	26
55	23	3	26
56	28	0	28
57	27	1	28
58	26	2	28
59	25	3	28

Table 2: Combinations of Doubles and Triples teams for a DOUBLES tournament with an even number of teams.

Similarly to before, these are not the only combinations, just the combinations with the maximum number of doubles teams. Other combinations can be obtained by decreasing L and using the same relationships as set out above.

Rod Deakin,
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